Bracing of slender steel and timber structures

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P.O. Box 118, SE-221 00 Lund, Sweden
Report TVBK-1051
ISBN 978-91-879930-6-0
ISRN LUTVDG/TVBK-1048/15-SE(258)
ISSN 0349-4969

Printed in Sweden by Media-Tryck, Lund University
Lund 2015

Preface

The work presented in the thesis was conducted by the author between 2013 and 2015 at the Division of Structural Engineering, Lund University.

My interest in structural stability started to grow during the snowy winters of 2009/10 and 2010/11 when hundreds of buildings in Sweden collapsed. It was a terrifying but also interesting time, there being new collapses to discuss at the coffee table every morning.

I would like to express my gratitude to my main supervisor, professor Roberto Crocetti, together with whom the basic ideas of the project were formulated. I am also grateful for the support given me by my assistant supervisors, Dr. Eva Frühwald Hansson, of Lund University, and Dr. Joakim Jeppsson, of the firm Skanska, and all of the members of the reference group. I would also like to thank Skanska Teknik for letting me work on this project and all members of the staff of the Division of Structural Engineering for the joyful time I have had.

The work presented in the thesis would not have been possible to carry out without the financial support of both SBUF and Skanska Sverige AB for which I am exceedingly grateful.

Finally, I would like to thank my family and friends for their support and patience.

Anders Klasson
Lund, November 2015

Abstract

During winters in which there has been a heavy load of snow, hundreds of slender roof structures have collapsed both in Sweden and in other countries. The main cause of failure here has been shown by forensic investigations to be insufficient bracing. These structural collapses, and others that occurred, have emphasized very much the importance of the subject of structural stability.

The study at hand investigates certain basic matters concerning the relationships between the bracing stiffness, bracing forces and buckling modes of slender braced structural members. It also examines the importance of imperfections and how these affect the expected buckling capacity of braced members, as well as the bracing forces involved. Also investigated here is the question of whether the slip which typically occurs in bracing systems has the potential of affecting appreciably the buckling capacity of braced members. The thesis begins by presenting a brief overview of some of the collapses that have occurred during snowy winters in Sweden as well as of some of the collapses that have occurred during the construction process.

The structural members that are analyzed within the scope of the thesis are braced timber beams, trusses and steel columns. The methods employed for the examinations conducted include laboratory testing (of timber beams), finite element analysis with the use of the commercial FE-program Abaqus and analytical studies (of columns) by means of the energy method.

The major conclusions drawn in the investigations conducted are the following:

- It has been verified that both the strength and the stiffness requirements of bracings need to be taken into account in order to achieve safe design.

- It is important to use adequate imperfection models in designing slender structural elements by means of non-linear analysis. Inappropriate choice of imperfection shapes can lead to totally unrealistic results.

- A superposition of different imperfection shapes should be used for conservative design of the bracing system and the braced member itself. Use of imperfection shapes of large magnitude at the points of the bracings lead not only to strong bracing forces but also to greater pre-buckling stiffness of the braced elements. The latter might lead to nonconservative design of the braced members.

- Slip in bracing systems has the potential of significantly reducing the load bearing capacity of braced members.

Papers

1. *Discrete bracing of timber beams subjected to gravity loads*
   Anders Klasson, Roberto Crocetti and Eva Frühwald Hansson
   World Conference on Timber Engineering, 2014

2. *Slender steel columns: how they are affected by imperfections and bracing stiffness*
   Anders Klasson, Roberto Crocetti and Eva Frühwald Hansson
   Structures, submitted for publication (2015)

3. *Stability of slender timber members: the consequences of potential slip in the bracing system*
   Anders Klasson, Roberto Crocetti and Eva Frühwald Hansson
   Journal of Structural Engineering, submitted for publication (2015)

Contents

1 Introduction

1.1 Background

In Sweden, a large number of roof structures (> 180 of them) collapsed during the winters of 2009/10 and 2010/11. The snow load during these winters was greater than usual but generally did not exceed the load limit specified by the code [1], this indicating clearly that the structures were inadequately designed. The majority of the structures that failed were of a slender type, made either of timber or steel. Such structures are known to be sensitive to different instability phenomena; forensic investigations point to insufficient bracing as being the major cause of the collapses that occurred during the winters in question [2].

According to a report [10] produced by SP, Skanska Sverige AB and Lund University, some 60% of the roofs that collapsed during 2009/10 and 2010/11 were built after 1980. It might thus appear that we have been building in a less accurate way since 1980 than before. However, as Johannesson et al. [9] point out, it occurred already during the winter of 1976/77 that there were structures that were damaged and a considerable amount that failed then as well. This means that some of the structures built before 1980 that would have had the potential to collapse during the winters of 2009/10 and 2010/11, had already collapsed that earlier winter. It can thus not be concluded that we have been building less accurately since 1980 than before, but rather that we have been building in a perhaps about equally defective way as had been done earlier, and that there are many newer buildings too that will be likely to collapse sometime in the future, if building practices are not improved.

Roofs collapsing due to insufficient bracing when loaded by snow has not only been a problem in Sweden. During the winter of 2005/06 more than 50 roofs failed in Germany, Austria and Poland [7], for example. Also here, as in the Swedish cases, it has been concluded that the snow loads that winter were large but generally did not exceed the limits as defined by codes. A Nordic study concerning 127 timber building failures around the world (with one exception these not including the collapses already mentioned) concludes that instability, i.e. use of insufficient bracing, is the most common cause of building failures in general.

Slender structures have not only been found to fail due to insufficient bracing when they have been loaded by heavy snow. According to Frühwald et al. [7], slender structures that fail, most of them trusses, often do so already during the construction phase. This reflects the fact that the importance of temporary bracing while buildings are being erected is often neglected and that the bracing system of the finalized building is normally not fully effective at this phase.

Since building collapses can lead to the loss of human life, as well as to economic losses, they can not be simply tolerated by society, building codes needing to be written in order to control how buildings are designed and how sufficient safety can be ensured. However, despite such code regulations, buildings do fail. The main reason for this can be that equally important, sometimes even more important than rules and regulations, are proper engineering and fundamental understanding of structures in general when designing them. Since understanding of this sort can probably not be adequately "specified" in a code of any sort, really competent engineers will always be required. At the same time, as the results of a licentiate thesis presented at Lund University [6] emphasize, the designing work of different engineers can also differ substantially in ways that may not immediately be obvious how they affect matters of safety, this probably being the case in many respects in regard to the designing of bracing as well.

The work of the present thesis started with an investigation of certain fundamental principles concerning the stability of slender structures, both in general and in regard to bracing, to endeavor to illuminate various considerations that are important for engineers to bear in mind designing structures in such a way that unexpected collapses of the sort described can be avoided in the future. It should be emphasized that the present thesis serves as an interim reporting (a licentiate thesis) of a project to be dealt with more comprehensively later in a doctoral thesis.

1.2 Objectives

The overall objective of the present thesis is to contribute to a more adequate understanding of the buckling and bracing requirements that are needed in regard to both slender steel and timber members. The more specific objectives were the following:

1. To discuss a number of collapses that have occurred due to instability, with the aim of demonstrating the significance of proper stabilization and bracing.
2. To investigate certain basic principles regarding relationships between bracing stiffness, bracing forces and buckling modes.
3. To investigate the effects of various imperfections in terms of how they affect the forces to be expected in the bracing system and the buckling capacity of the braced members in question.
4. To investigate how and to what extent potential slips in bracing systems can reduce the expected buckling capacity of braced members.

1.3 Limitations

The investigations are limited to the study of individual structural members. These include discretely braced glued laminated beams, steel columns and a three-hinged timber truss. Only discrete translational bracing is dealt with in the thesis.

1.4 New findings

The main findings of the thesis appear to be the following:

1. A superposition of different imperfection shapes should be used for conservative designing of the bracing system and of the braced elements.

2. Various imperfection shapes can be shown to generate unrealistic behavior when non-linear analyses are performed. It is important, therefore, to make use of realistic imperfection models.

3. Slip that occurs in bracing systems can have a significant effect on the buckling strength of a braced structural member.

4. Bracing forces of stocky members, or members loaded to a degree substantially smaller than the critical load, increase with increasing bracing stiffness whereas, as known before, the opposite holds for slender members.

1.5 Outline of the thesis

The contents of the thesis have been chosen in order to consolidate, support and broaden the contents of the appended papers.

- Section 1 of the thesis presents certain background information concerning the project.

- Section 2 presents various examples of collapses that have occurred, both during the construction and during the service life of different structures.

- Section 3 present primarily certain information regarding bracing so as to extend the information given in the appended papers.

- Section 4 provides a summary of the appended papers.

- Section 5 summarizes the conclusions of both the thesis as a whole and of the appended papers.

- Section 6 discusses further research in line with this that can be suggested.

The thesis contains three appended papers. These deal with different concepts regarding the stability of slender steel and timber members, respectively, in line with the objectives of the thesis. The first paper was presented at the *World Conference on Timber Engineering*, WCTE, in Quebec City in 2014. The other two papers have been submitted for publication in scientific journals.

2 Failures due to instability

In this chapter a number of examples of buildings that have collapsed (also reported on in literature in this area generally) are provided. The aim of reporting them here again is to show that the mistakes that lead to structural failures can be seen as basically the same or very similar over time, suggesting that one does not appear to learn sufficiently from experience. Section 2.1 takes up examples of different collapses that occurred during the winter of 1976/77. Section 2.2 provides examples of various collapses that occurred during the winters of 2009/10 and 2010/11. Section 2.3, in turn, describes various collapses that have occurred during the construction phase.

2.1 Snow winter of 1976/77

The collapses that occurred during the winter of 1976/77 were located in the same regions as those of 2009/10 and 2010/11 [10]. According to Johannesson et al. [9], the snow loads were higher than those referred to in the code in only 12 out of 94 cases. Most of the failed structures were slender steel or timber structures, i.e. structures of the same type as those that failed during the winters 2009/10 and 2010/11. Three different collapses, similar to those reported in the collapses of 2009/10 and 2010/11, are presented below.

Figure 2.1 shows a collapsed riding hall. The structure was a frame built up of lattice elements. The bracing on the ”legs” of the frames appears to be quite adequate. In the corners of the frames, however, the inside is left completely unbraced. This reported to be the primary reason for the collapse of this building. Note the similarity to the collapse presented in Figure 2.5 which occurred more than 30 years later.

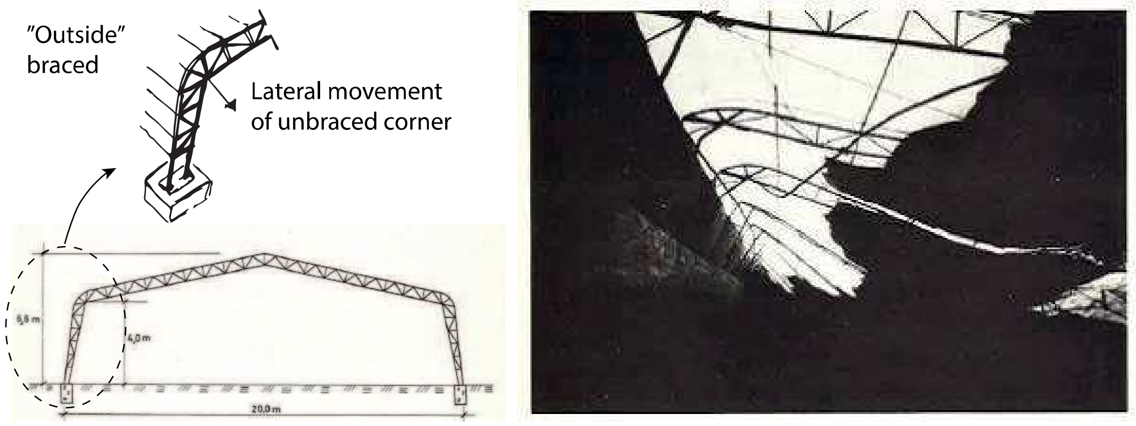

Figure 2.1: *A riding hall built in 1975. Note the buckling of the unbraced corner. (Johannesson et al. [9])*

Figure 2.2 shows a collapsed storehouse. The inside portions of the frames are left unbraced in critical parts in which the moments are large, namely those close to the corners. This collapse is also similar to those shown in Figures 2.1 and 2.4. It appears to be a common mistake to assume that the bracing provided by walls, connected to only one of the flanges is adequate to prevent buckling, this collapse clearly indicates the opposite to be the case.

Figure 2.2: *A storage building built in 1974. The columns are unbraced on the parts insides where they buckled. (Johannesson et al. [9])*

Figure 2.3 shows a collapsed canopy roof. Bracing there is lacking, especially in the regions in which the main girder is compressed on its bottom side, i.e. over the columns. According to Johannesson et al. [9], lateral torsional buckling was initiated in that region due to the lack of bracing. Note the similarity to the collapse from the winters of 2009/10 or 2010/11 shown in Figure 2.4. In both cases, bracing of the compressed side of the beam above the supports was omitted.

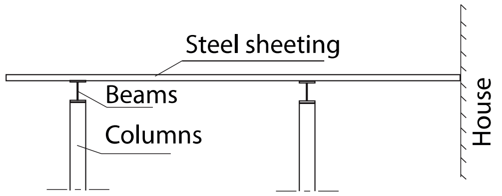

Figure 2.3: *Canopies built in 1975. The primary beam is left unbraced on the compressed side above the supports (columns). The beam probably failed due to lateral torsional buckling. It appears as though thereafter the columns buckled at their tops as a consequence of the large deformations that occurred. (Johannesson et al. [9])*

2.2 Snow winters of 2009/10 and 2010/11

As reported in certain forensic investigations [2, 10], the snow measured on flat surfaces during the winters of 2009/10 and 2010/11 did not in most cases exceed the design values specified by the code. Several potential explanations for these collapses are given. These include snow drift not taken account for properly in the design, ignorance regarding the stabilization requirements that apply, and carelessness in general regarding both design and construction. The most important cause of collapses is reported to be insufficient bracing. Two examples of collapses brought about by insufficient bracing are given below.

Figure 2.4 shows both a sketch and a picture of a riding hall that collapsed during one of the winters in question. Purlins were used to stabilize the main load-bearing elements, i.e. the beams, against lateral torsional buckling. However, as can be clearly seen in both the sketch and the picture of the actual building, there is no bracing in the compressed parts above the supports (columns) of the beam. It is well known that bracing on the tension side of a beam has only minimal effects on the buckling capacity. Thus, the main explanation for the collapse of this building (as was also indicated in a master thesis at Lund University [18]), is probably lateral torsional buckling of the primary load bearing beam being initiated above the columns.

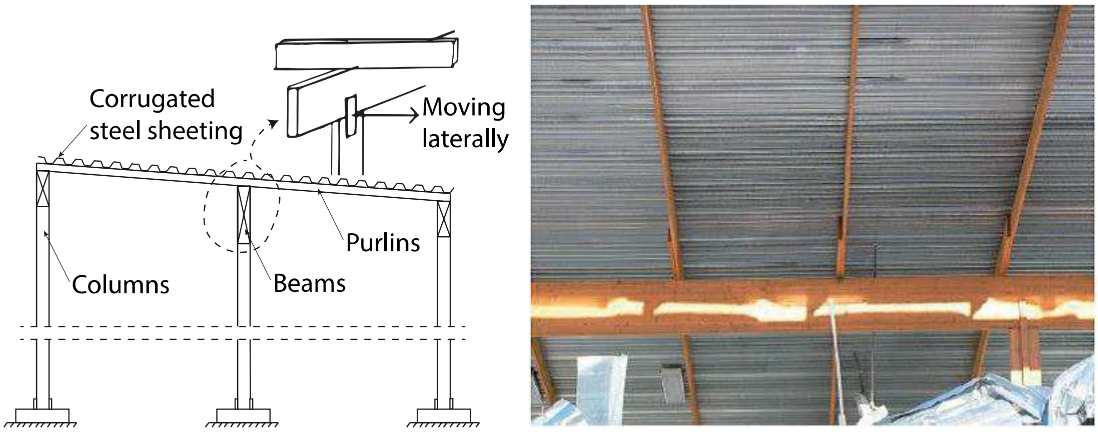

Figure 2.4: *Riding hall. Bracing is lacking on the compressed side of the beams over the supports (columns).(Johansson et al. [10])*

Figure 2.5 shows the collapse of a steel frame. It can be clearly seen in the figure that the collapse was initiated at the corners of the frame. It can also be seen that bracing of the compressed side of the corner had been omitted. This, i.e. inadequate bracing, was most likely the primary reason for the collapse.

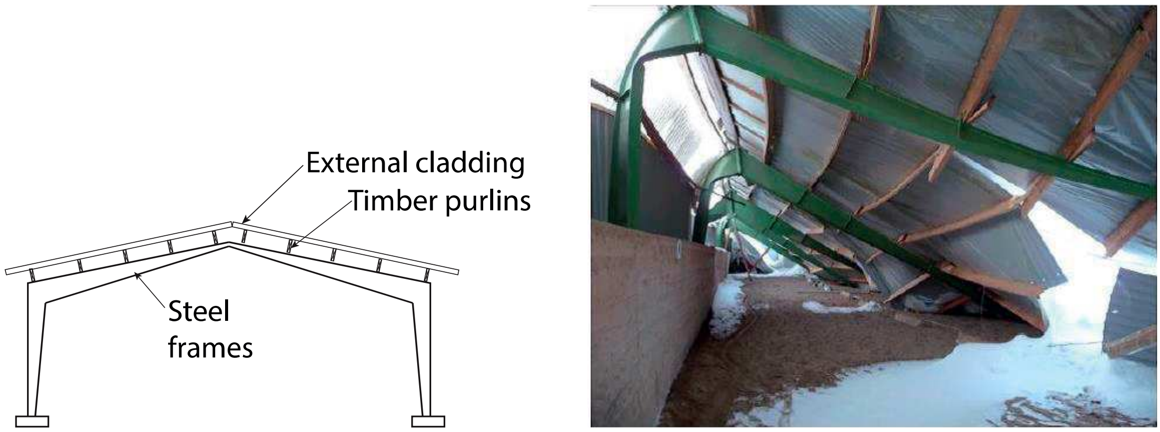

Figure 2.5: *Riding hall. Bracing is lacking on the compressed side of the corner, which as a result beams got instable and buckled. (Johansson et al. [10])*

2.3 Collapses during construction

Typically, structures are most vulnerable during the construction phase. The main reason for this vulnerability is that the bracing system is normally not fully effective in this phase. Two different examples are given below.

Figure 2.6 shows a steel/concrete composite bridge, the Marcy bridge (New York City), that collapsed during concreting of the deck. In its finalized state, i.e. after hardening of the concrete had taken place, the bridge would have been stable due to sufficient torsional stiffness of the cross section then. Before hardening of the concrete, however, the cross section was open and was thus utterly sensitive to lateral torsional buckling due to the low degree of torsional stiffness it possessed. The bridge failed due to lateral torsional buckling when casting of the bridge deck reached the mid-span of the bridge. In order to have prevented such a failure, as described in greater detail by Mehri et al. [12], more adequate temporary cross bracings would have been required during the casting of the bridge deck. Further collapses of this type are compiled in a doctoral thesis [11], including the bridge Y1504 that collapsed in Sweden during the summer of 2002.

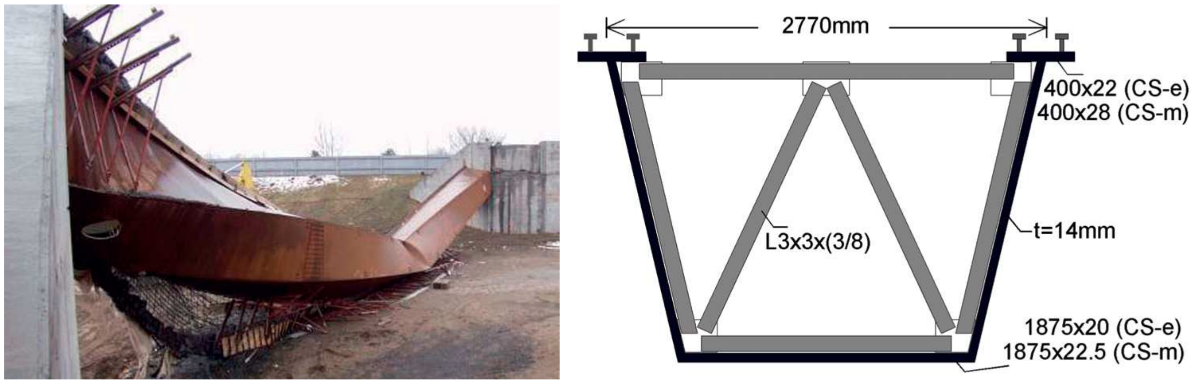

Figure 2.6: *The Marcy Bridge in New York City that collapsed due to insufficient bracing when concreting of the bridge deck reached mid-span. a) A picture of the collapsed bridge. b) The cross section of the bridge. (Mehri et al. [12])*

One "famous" example of a building that failed during construction is the Rosemont Horizon Arena (Chicago, USA), that collapsed during its erection in 1979, see Figure 2.7. The roof was built of timber arches that were stabilized by purlins. However, the purlins themselves were not connected to a stabilized bay; no bay at all was actually stabilized in the structure, which means that there was no place where the forces in the purlins could be transferred to the ground and thus stabilize the arches. In its finalized state, there would have been a steel sheeting on top of the structure that would have stabilized the roof through diaphragm action. During the construction phase, a temporary stabilization system would have been required in order to prevent the collapse. The collapse was initiated by a small wind load in the direction perpendicular to the arches.

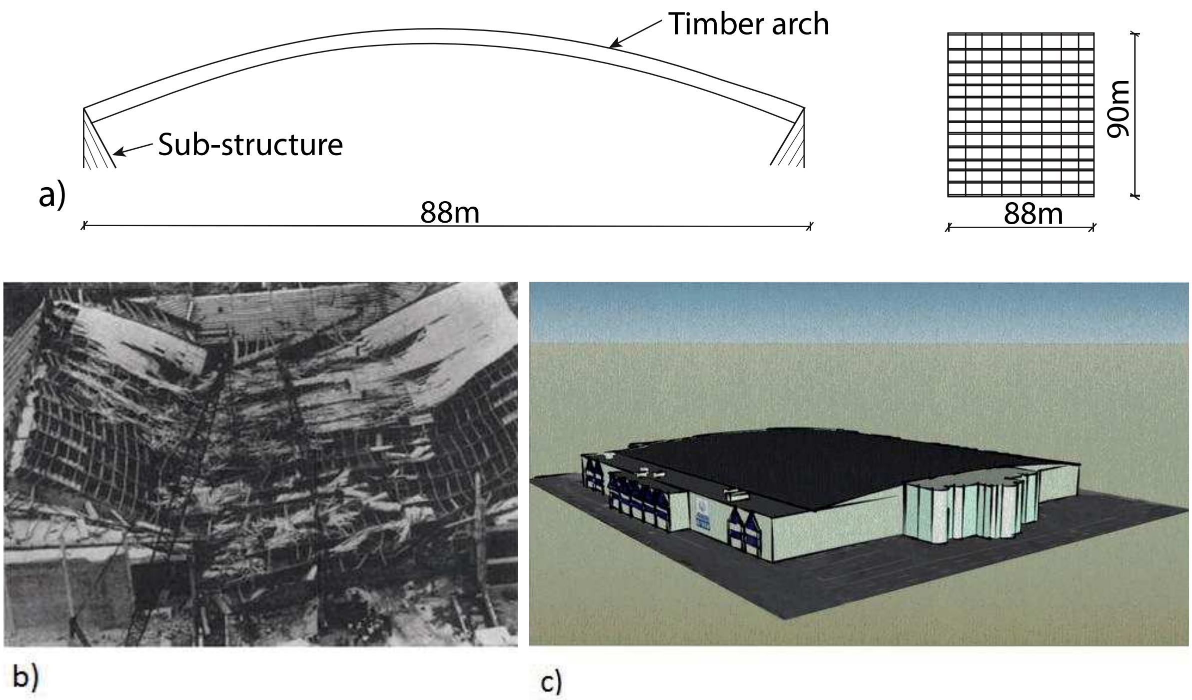

Figure 2.7: *The collapse of the Rosemont Horizon Arena, USA. a) A sketch of the structure. Note that there is no stabilizing bay. b) A picture taken after the collapse. c) What the building looked like after finalization (it was rebuilt after the collapse). (All images in public domain - downloaded from https://failures.wikispaces.com)*

3 Bracing and Buckling

The concept of the bracing of a structure, as defined here, involves not only distinguishing a stable state from an unstable one. Bracing is also a relative concept, since the properties of bracing can be varied in order to ensure that the stability is sufficient in a given situation. This means that a structure can be provided an amount of bracing sufficient to ensure that a specific level of resistance to different instability phenomena that can develop is attained.

According to Galmbos [8], in contrast to the statements in the paragraph above, bracings are commonly considered to be ideal by engineers when designing structures. With ideal bracing is meant that the bracings are assumed to provide perfect restraint to the structural members that are being braced, this normally meaning that no lateral displacements will take place at the bracing points. This, of course, is neither possible nor necessary in most real structures.

Bracing of structures can be said to be required in two steps, the first of these being compulsory for most structures:

1. Brace the structure so that it is statically stable, i.e. has at least one way to carry an arbitrary load, as exemplified by step 1 in Figure 3.1.

2. Regarding buckling, brace the structure so as to ensure that it does not become unstable (does not buckle) in response to a minimum specified load. This holds both for a structure in its entirety (global buckling), as well as for the stability of separate parts of the structure (e.g. the beams in a particular roof structure). Step 2 in Figure 3.1 exemplifies additional bracing that is provided so as to enhance the in-plane buckling capacity of the vertical members in the frame.

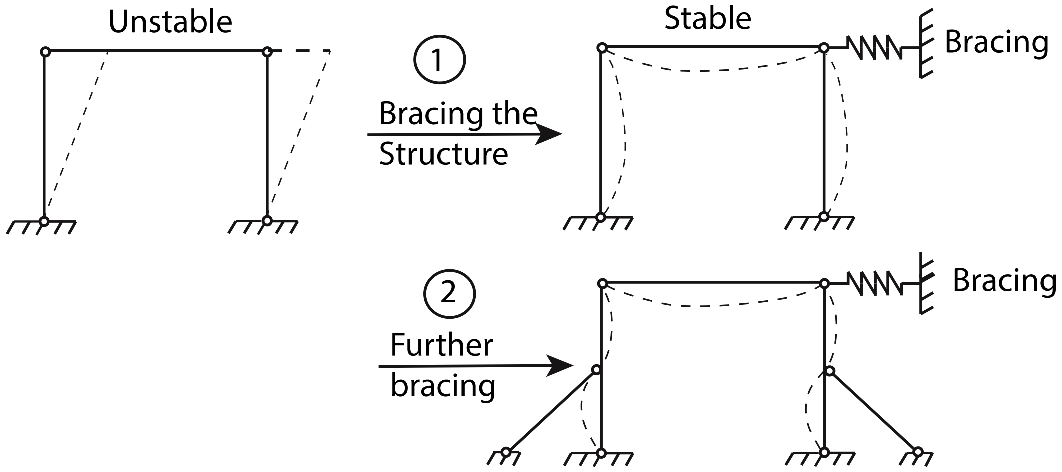

Figure 3.1: *The in-plane bracing of a frame.*

The literature (e.g. [8] and [21]) defines primarily bracing methods of four different kinds, namely discrete bracing, continuous bracing, relative bracing and lean-on bracing [8, 21]. Since the investigations carried out in the present work (see the appended papers) dealt with the discrete bracing of beams (papers 1 and 3), of columns (Paper 2) and of trusses (Paper 3), with different aspects, of these being considered, more detailed information regarding this bracing method is given in the following sections.

In addition, bracing, especially that of the discrete and of the continuous type, can be either torsional or translational, or in some cases a combination of the two. Only translational bracing is dealt with in the thesis.

3.1 "Idealization" of buildings

A limitation of the work presented in the thesis was the study of individual structural elements. For that reason, a short discussion of how the rest of the parts in a building can be replaced by equivalent bracings is provided here.

At the start, how a hall-type building can be stabilized against wind loading is considered. Figure 3.2 shows the principles that apply when stabilization is achieved by means of trusses in the vertical and horizontal planes of the building. Another method of accomplishing stability would be to use a steel sheeting that can ensure stability by means of diaphragm action (not shown here). The same system, when used for wind stabilization, is normally also used for restraining or bracing the primary load-bearing elements (e.g. beams, trusses, arches or frames) from buckling.

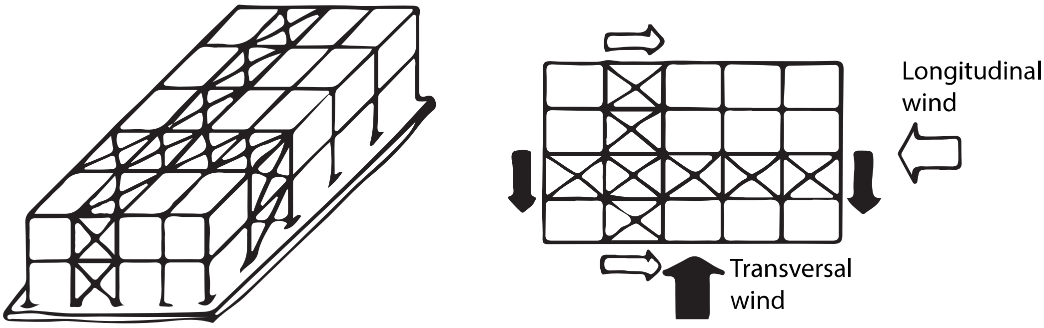

Figure 3.2: *Wind bracing of a hall-type building*

In order to estimate the stiffness of an equivalent bracing that is to be used in designing the different parts of a load-bearing structure individually, all the parts from the point of bracing of the member to where the forces involved reach the foundations have to be considered. Usually, it is the bracing system that resists longitudinal wind forces (see Figure 3.2) that also provides the primary load-bearing elements with buckling capacity, see Figure 3.3. The stiffnesses of the different parts included in the expression of the equivalent stiffness can be found by exposing them to a unit load in the bracing direction and noting the deflection that occurs at that point. The stiffness of that part, at that particular point, is then equal to the applied force divided by the displacement obtained at that same point.

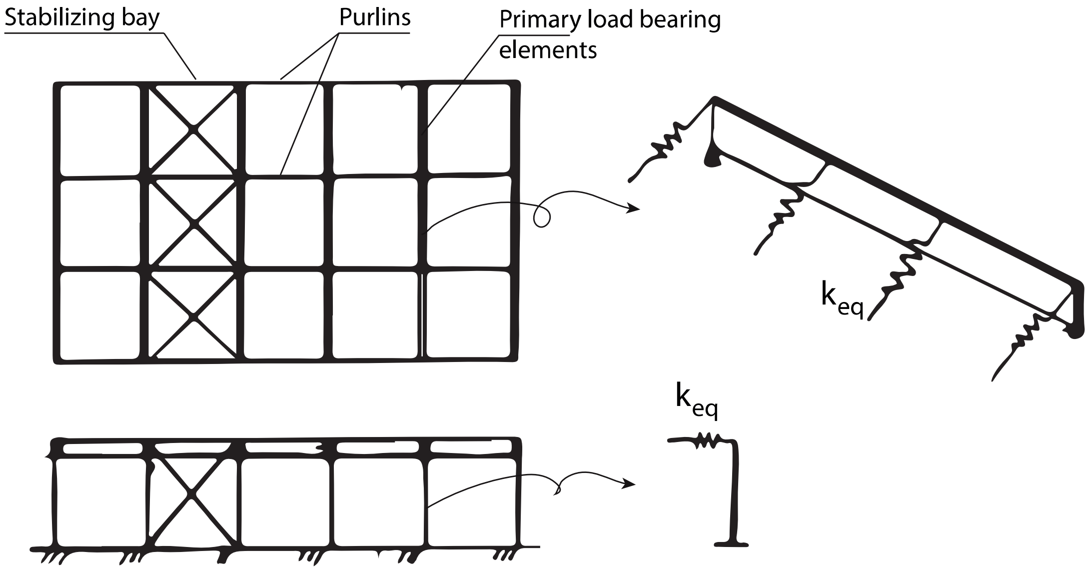

Figure 3.3: *How longitudinal wind bracing systems act as restraints for the different primary load bearing elements.*

The stiffness of the equivalent bracing, denoted as k_{eq} in Figure 3.3, is inversely proportional to the stiffness of the different parts of the bracing system, as shown in Equation 3.1 (which is a well known expression for serial springs).

$$\frac{1}{k_{eq}} = \frac{1}{k_{purlin}} + \frac{1}{k_{stabilizing-bay}} \tag{3.1}$$

The accuracy of the stiffness estimation of the bracing system (Equation 3.1) can be improved by considering the bracing system on a more detailed level, for instance by also including the stiffness of potential connections ($1/k_{connection}$) along the purlins in the expression. In that case, the equivalent bracing stiffness is given in terms of Equation 3.2.

$$\frac{1}{k_{eq}} = \frac{1}{k_{purlin}} + \frac{1}{k_{stabilizing-bay}} + \frac{1}{k_{connections}} \tag{3.2}$$

In Paper 3, the potential consequences of slip in the bracing systems of both a timber beam and a truss were investigated. A few additional remarks concerning potential sources of such slip and/or of very low stiffness at the beginning of loading than what was given in the paper are presented here (mostly valid for timber structures). These effects are also illustrated in Figure 3.5.

1. The most important potential source of slip in purlins is very likely the connections involved, especially bolted ones, as was shown by Dorn et al. [5] to be the case. The slip in question can for instance be related to over-sized holes.

2. Another type of slip, defined here instead as a movement that occurs with a low degree of stiffness, can be the initial crookedness of the purlins working in tension. That means that the purlins need to straighten out a bit before they are able to resist loading to the intended degree; see Figure 3.4. This can be compared to the action of a cable, the stiffness of which is zero when it is slack.

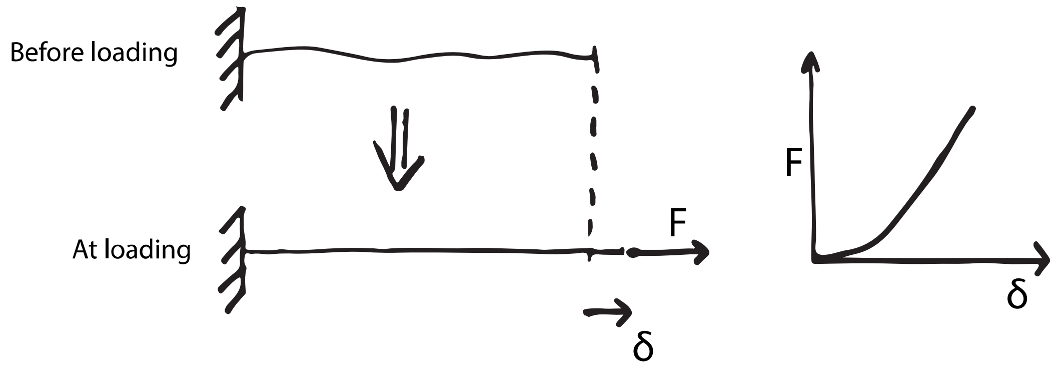

Figure 3.4: *An illustration of how initial crookedness can prevent a bracing system from resisting load directly after it is applied.*

3. Still another type of slip is the relaxation of the cables used in the stabilizing bay as well as in connection with creep of the timber in the stabilizing bay. In order for the stabilizing bay to immediately resist lateral loading, the cables

need to have a prestress to some degree. Over time, such prestress usually becomes reduced in size due to relaxation. In addition, if the cables are highly stressed, the timber can tends to creep in that direction and thus to contribute to the slacking of the stabilizing bay.

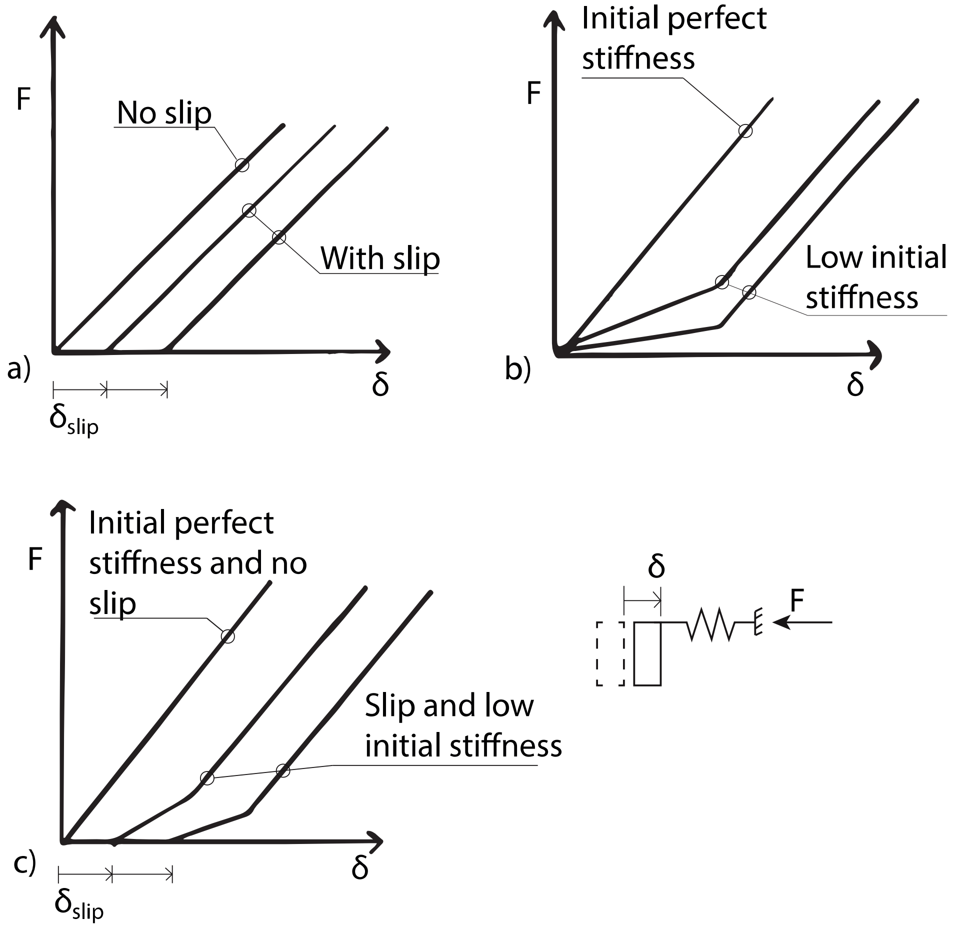

Figure 3.5: *a) The response in a bracing system that require a certain slip δ_{slip} prior to its being activated. b) The response in a bracing system that is of low initial stiffness. c) The response in a bracing system that has both slip and low initial stiffness.*

3.2 Columns

In order to introduce the concept of ideal stiffness, a simple example is provided below.

The most simple case of column bracing is probably that of a single column that is pinned at its base and is supported by a translational spring (bracing) at its top; see Figure 3.6. Such a column can basically buckle in two different modes, namely as i) a rigid bar (Figure 3.6a) if the bracing is weak, and ii) as an Euler 2 column (Figure 3.6b) if the bracing is strong (i.e. sufficiently stiff).

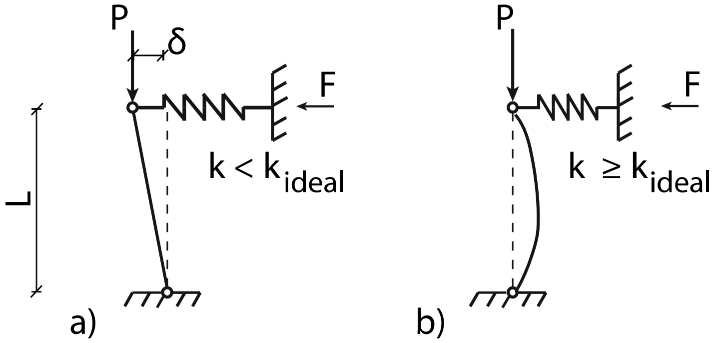

Figure 3.6: *Two different buckling modes of a simple column. a) Weak bracing resulting in a pendulum buckling mode. b) Strong bracing forcing the column to buckle in a sine-shaped mode.*

The buckling load P of the column presented in Figure 3.6a can be calculated on the basis of equilibrium considerations (small displacement theory), in accordance with Equations 3.3-3.4 below.

The moment equilibrium about the base point of the column:

$$M_{basepoint} = P\delta - FL = 0 \tag{3.3}$$

where P is the buckling load, δ the displacement at the top, F the force that occurs in the spring and L the length of the column.

Inserting $F = k\delta$ (force = bracing stiffness x the displacement) in Equation 3.3 and equating for P results in Equation 3.4.

$$P = kL \tag{3.4}$$

where k is the stiffness of the bracing.

If the limit of stiffness k_{ideal} that enables the column to buckle in a half sine wave (Euler 2), as shown in Figure 3.6b, is of interest to calculate, one can note that it is simply to equal the expression in Equation 3.4 to the Euler 2 buckling load, one's solving then for k as shown in Equation 3.5.

$$P = kL = \frac{EI\pi^2}{L^2} \Rightarrow k_{ideal} = \frac{EI\pi^2}{L^3} \tag{3.5}$$

where EI is the bending stiffness of the column in question.

In order to rewrite in terms of the buckling load P of the column, for all values of k, P is obtained in accordance with Equation 3.6. A plot of Equation 3.6 is shown in Figure 3.7. In this example, one can see that the ideal stiffness is simply defined as the stiffness for which an increase in stiffness would not generate an additional (elastic) buckling capacity.

$$P = \begin{cases} kL, & \text{if } k < k_{ideal} \\ \frac{EI\pi^2}{L^2}, & \text{if } k \geq k_{ideal} \end{cases} \tag{3.6}$$

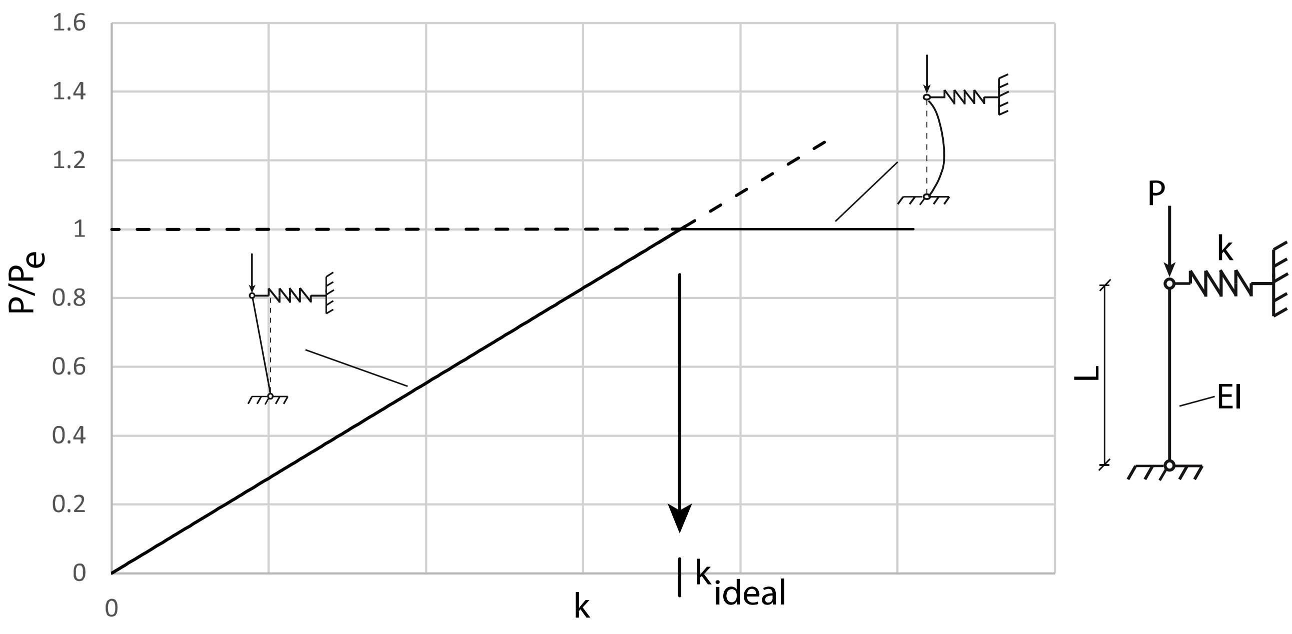

Figure 3.7: *A plot of Equation 3.6, where P is the buckling load and P_e is the Euler 2 buckling load. The ideal stiffness k_{ideal}, i.e. the stiffness for which an increase in bracing stiffness would not generate a higher degree of buckling capacity, is indicated by the arrow in the figure.*

For more complicated systems, such as columns with several intermediate bracings, it is not possible to obtain the ideal stiffness on the basis of equilibrium considerations only (if closed form solutions exist at all). For instance, Trahair [17], Al-Shawi [3] and Plaut et al. [14, 15], have all done important analytical work (such as involving kinematics and/or constitutive relations) on solving the problem of a column pinned at both ends having one or more intermediate bracing. These solutions (not shown here) are rather involved and for more complicated systems they may not even exist.

Rigid link model

Due to the complexity of the solutions to the problem at hand, as described in the section above, approximate solutions are quite in order. In 1958, Winter [19] presented a simple yet powerful rigid link method that can be used in designing any pinned columns having an arbitrary number of brace points. The method can be used both for calculating the ideal stiffness as for taking the strength of the bracings into consideration in determining the adequate bracing dimensions. The method is briefly explained below.

In using the rigid link method, one assumes the bracings to be perfectly effective, and places fictitious links in these points. Thereafter, by assuming there to be an Euler 2 buckling mode between successive bracings, i.e. the maximum axial load that the system can possibly withstand, equilibrium considerations can be used then to calculate the required stiffness, k_{ideal}, for obtaining the maximum buckling capacity in question. In order to study this equilibrium, a fictitious (infinitesimal) perturbation has to be given to the system, i.e. a very small displacement at the bracing point, so as to be able to obtain an expression for the bracing force. From a moment equilibrium about, for instance, one of the links, an expression for the ideal stiffness can now be readily calculated; see Figure 3.8 for an example of a simple pinned column having one intermediate bracing.

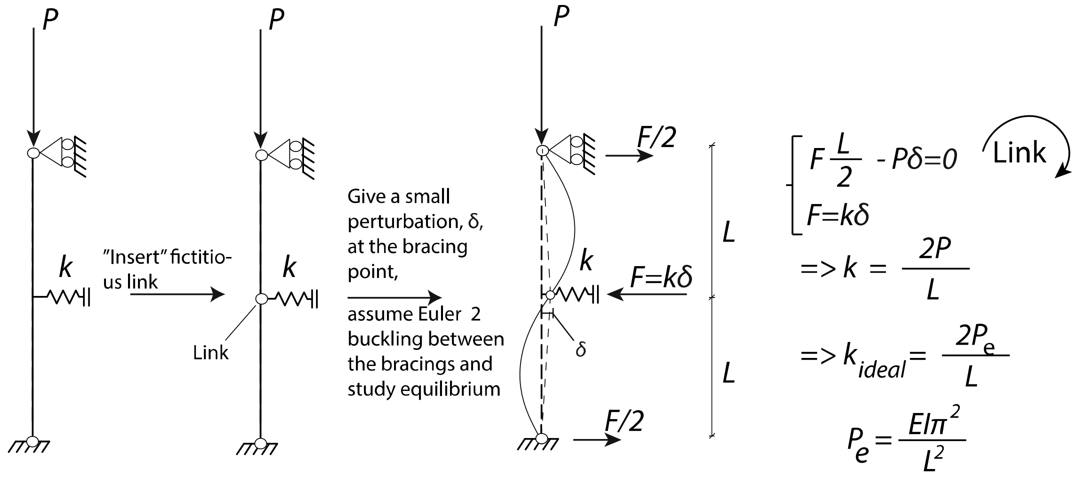

Figure 3.8: *The rigid link model for a column pinned at both ends having one intermediate bracing. P_e is the Euler 2 buckling load, F the force in the bracing, δ the assumed displacement of the bracing and k_{ideal} the ideal bracing stiffness of the system in question.*

In order to be able to study the strength requirements of bracings, Winter [19] introduced an initial imperfection, δ_0, into the solution of his rigid link model. This means that the total displacement at the bracing point is $(\delta_0 + \delta)$. In solving the example shown in Figure 3.8, in which there is an initial imperfection of δ_0, moment equilibrium about the link, is obtained by the use of Equation 3.7:

$$\frac{Lk\delta}{2} = P(\delta_0 + \delta) \Leftrightarrow k = \frac{2P}{L}(\frac{\delta_0}{\delta} + 1) \tag{3.7}$$

Substituting $\delta_t = \delta_0 + \delta$ (where δ_t is the total displacement) into Equation 3.7 and noting that $k_{ideal} = \frac{2P_e}{L}$, solving for δ_t gives the expression shown in Equation 3.8.

$$\delta_t = \frac{\delta_0}{1 - \frac{2P}{kL}} = [k = k_{ideal}] = \frac{\delta_0}{1 - \frac{P}{P_e}} \tag{3.8}$$

where P_e is the Euler buckling load of the column in question.The applied force P at the normalized bracing displacement δ_t/δ_0 is plotted in Figure 3.9a.

The force F in the bracing can be found through combining Equation 3.7 with the relationship between the bracing stiffness, the bracing force and the displacement that the bracing results in, according to Equation 3.9.

$$F = k\delta = \frac{2P}{L}(\frac{\delta_0}{\delta} + 1)\delta = \frac{2P}{L}(\delta_0 + \delta) \tag{3.9}$$

Assuming ideal stiffness, k_{ideal}, and combining Equation 3.9 with Equation 3.8, the expression of F can be obtained in accordance with Equation 3.10. Note that the equation is only valid for $F/P > 2\delta_0/L$.

$$F = \frac{2P}{L}\frac{\delta_0}{1 - \frac{P}{P_e}} \Leftrightarrow \frac{P}{P_e} = 1 - \frac{1}{F/P}\frac{2\delta_0}{L} \tag{3.10}$$

The normalized bracing force F/P is plotted against the normalized applied load P/P_e (Equation 3.10) in Figure 3.9b. The magnitude of the bracing force is dependent, however, on the magnitude of the initial imperfection. In the plotting carried out, the magnitude of the initial imperfection was chosen to be $L/500$.

If the bracing stiffness is two or three times the ideal stiffness, the relationship between P/P_e would instead be according to Equations 3.11 and 3.12, respectively. These relations also are plotted in Figure 3.9. The equations are only valid for $F/P > 2\delta_0/L$.

For two times the ideal stiffness:

$$\frac{P}{P_e} = 2 - \frac{4}{F/P}\frac{\delta_0}{L} \tag{3.11}$$

where P is the load applied and P_e the Euler 2 buckling load between successive restraints, F the force in the bracing and δ_0 the initial imperfection.

For the case of three times the ideal stiffness:

$$\frac{P}{P_e} = 3 - \frac{6}{F/P}\frac{\delta_0}{L} \tag{3.12}$$

where P is the load applied and P_e the Euler 2 buckling load between successive restraints, F the force in the bracing and δ_0 the initial imperfection.

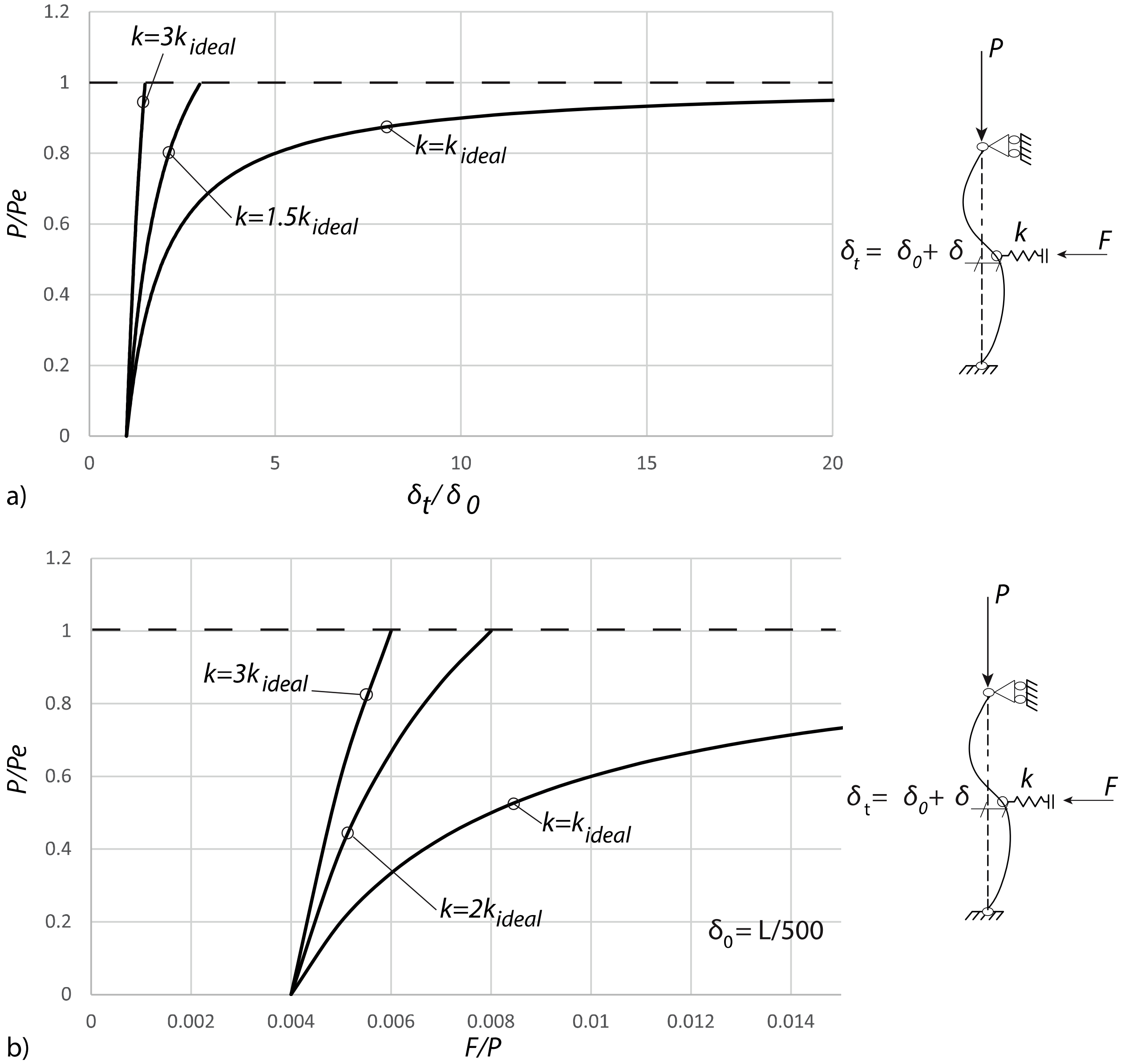

Figure 3.9: *The rigid link model for a column pinned at both ends with one intermediate bracing. a) The displacement at the bracing point for a load level of P for different bracing stiffnesses k. b) The bracing force as a function of P for different bracing stiffnesses.* P_e *is the Euler 2 buckling load, F the force in the bracing,* δ *the displacement in the bracing,* δ_0 *the initial displacement at the bracing point,* δ_t *the total displacement at the bracing point and* k_{ideal} *the ideal stiffness of the bracing in question.*

Figure 3.9 illustrates two of the most important features of bracing, namely i) if the bracing is equal to the ideal stiffness, the displacements in the bracing will be "infinitely" large as the buckling load is approached, and ii) to keep the bracing forces at reasonable levels, the stiffness of the bracing should at least be 2 times the ideal one.

Yura [20] later developed Winter's model to also hold for bracing stiffnesses of less than the ideal stiffness. That approach, not presented here, is particularly usable when there are multiple intermediate bracings, meaning that the buckling shape does not necessarily need to be described in terms of there being one half sine wave between successive bracings.

Finally, regarding the rigid link model, it should be emphasized that it is both an approximate and a conservative model. It is conservative since it neglects the bending stiffness contribution of the column itself by its putting frictionless joints at the brace points. For the special case of evenly distributed bracings, however, when studying the perfect column, i.e. without initial imperfections, the method is more or less "exact" since the displacements at bracing points would be zero at buckling, the real buckling load being exactly the one of Euler 2 buckling between successive bracings. For uneven spans, however, the rigid link model, in certain cases, underestimates the buckling capacity considerably, as discussed, for example, in a paper by Mehri et al. [13].

Energy approach

The energy concept was used in the thesis in order to be able to study more complex systems without having to neglect the stiffness contribution of the column, such as one needs to do in the case of the rigid link method, and in order to provide a tool for validating the results of FE-analyses. The new refinement of the energy method that was developed in the thesis is to make use of a shape function that can describe many different buckling shapes. This is done by means of an ordinary sine-function that contains a factor that controls how many waves are generated over a specific length, in this case the column length. This is described further later.

The energy method is based on the idea of there being a balance between the loss of potential energy, ΔT, and the gain of internal energy, ΔU at buckling, i.e that $\Delta T = \Delta U$; see e.g. Timoshenko et al. [16]. In the case of column buckling, the loss of potential energy is derived from the vertical movement of the applied axial load, P, such that $\Delta T = P\delta_x$, where δ_x is the vertical shortening of the column in question. The gain in internal energy in the case of braced column buckling consists both of the increase in energy of the bracings and the strain energy required to deform the column into a given shape (usually a sine-shaped one).

Paper 2 presents bracing and imperfection studies of two different columns; see Figure 3.10. The first column was sway-prevented and was equipped with one intermediate bracing. The second column was sway-permitted and was equipped with two bracings: one at its top and one at its midpoint. The energy method was used to derive the theoretical elastic buckling capacity of the two columns under the assumption of sine-shaped buckling modes; see Equations 3.14 and 3.13, respectively. The equations here, however, are a bit more general than those presented in Paper 2. This because they are valid for an arbitrary number of evenly distributed bracings, controlled by the parameter n, of the same stiffness k.

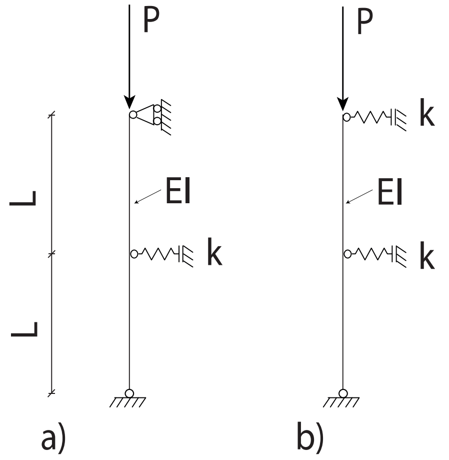

Figure 3.10: *Column systems 1 and 2 that were studied in Paper 2. a) System 1 b) System 2*

$$P^{syst1} = \frac{1}{4}\frac{EI\pi^4c^4 + 32\sum_{i=1}^{n}(sin^2(\frac{2\pi Li}{2L(n+1)}))}{L^2\pi^2c^2} \tag{3.13}$$

and

$$P^{syst2} = \frac{4kc^4L^3\sum_{i=1}^{n}(sin^2(\frac{2i\pi}{nc})) - EI\pi^3(sin(4\frac{\pi}{c})c + 4\pi)}{L^2\pi c^2(4\pi + csin(4\frac{\pi}{c}))} \tag{3.14}$$

where n is the number of evenly distributed bracings, c a shape factor (in the expression $sin(\frac{\pi x}{cL})$), L half the column length, k the spring stiffness, E the modulus of elasticity and I the moment of inertia.

There are, of course, a vast number of other column systems that could be of interest to analyze in the same manner as was done for System 1 and 2, but not all can be dealt with in the thesis. Examples of variations of the systems that are studied could be achieved by use of differing end conditions, such as rigid or semi-rigid, unequally spaced bracings, columns with non-constant EI, and both horizontal and vertical loading and bracings with different stiffness values. However, in order to demonstrate the refined method developed in the thesis, one additional system will be analyzed. The system can be considered as being common among real structures and will be referred to here as System 3.

System 3 is a column with a number of bracings having different stiffness values that have a non-linear distribution, derived in this case from the deflection formula of a cantilever beam, shear deformations being neglected as a simplification. Figure 3.11 exemplifies System 3 for the case of four bracings. The buckling shape of the column is assumed to be described by the function $\eta(x) = \Delta_\eta sin(\frac{\pi x}{cL})$. This function, as shown in Figure 3.12, can describe many different buckling modes of the column. A load, of P is assumed to be acting on the column at each floor level, meaning that the accumulated load on the column on the highest floor is 1P, on the second highest floor level is 2P and so forth.

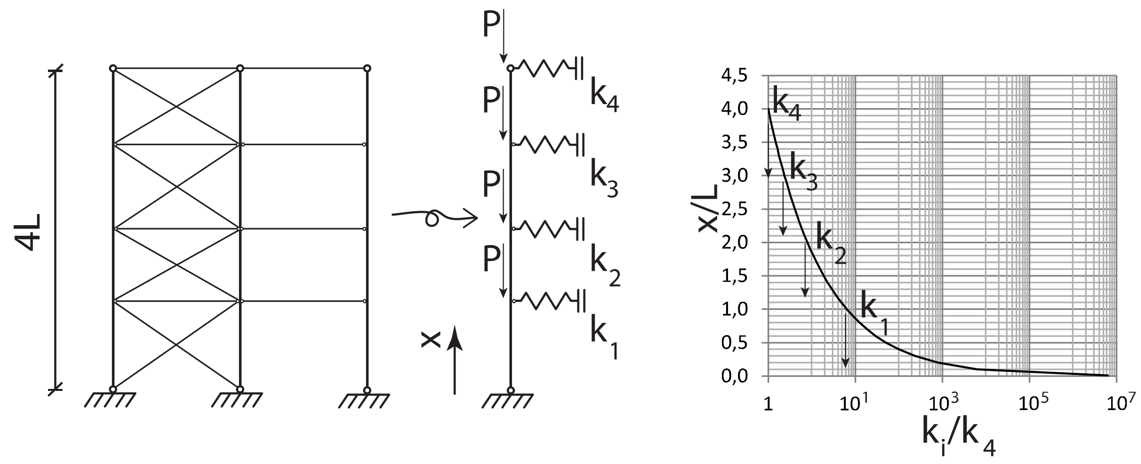

Figure 3.11: *Column System 3*

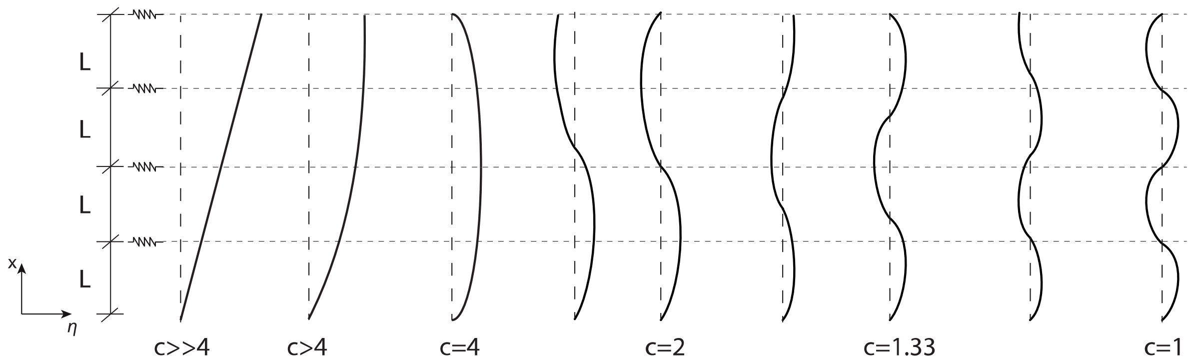

Figure 3.12: *Potential buckling modes for System 3, each described by the shape formula* $\eta(x) = \Delta_\eta sin(\frac{\pi x}{cL})$, Δ_η *being the magnitude of the function. Depending upon the c factor, which is selected, the buckling shape of the column can be everything from a straight bar (c>>4) to a half sine wave between successive bracings (c=1), as indicated in the figure.*

For the generalized case of System 3, i.e. there being arbitrary numbers of bracings and arbitrary stiffness values, an energy solution of the buckling load, P, was derived in accordance with Equations 3.15-3.19 below.

The first step is to calculate the loss of potential energy ΔT:

$$\Delta T = \sum_{i=1}^{n}(P\Delta_x^i) = \frac{P}{2}\sum_{i=1}^{n}\int_0^{iL}(\eta'(x)^2 dx) \tag{3.15}$$

where P is the axial force acting on each floor level, Δ_x^i the vertical shortening of the column on the i:th floor level, n the number of successive bracings, L the distance between the bracings and $\eta(x)$ the buckling mode shape. For further information regarding the integral describing the vertical shortening, see e.g. Timoshenko [16].

The second step is to calculate the gain in internal energy, ΔU, in the column:

$$\Delta U_{column} = \frac{EI}{2}\int_0^{nL}(\eta''(x)^2dx) \tag{3.16}$$

where EI is the bending stiffness of the column (assumed constant), n the number of bracings and $\eta(x)$ the buckling mode shape.

The third step is to calculate the energy gain in the bracings:

$$\Delta U_{bracings} = \frac{1}{2}\sum_0^n k_i\eta(iL)^2 \tag{3.17}$$

where k_i is the stiffness of the i:th bracing, $\eta(iL)$ the horizontal displacement at the i:th bracing and n the number of bracings.

The fourth step is to equate all the lost energy (ΔT) to all the gained energy ($\Delta U_{column} + \Delta U_{bracings}$) and to check the equilibrium, i.e. to assume that the gain of energy is equal to the lost energy:

$$\Delta T - \Delta U_{column} - \Delta U_{bracings} = 0 \tag{3.18}$$

The final step is to solve Equation 3.18 with respect to the buckling load P:

$$\begin{aligned}\Rightarrow P^{syst3} =& 2\frac{\frac{EI}{2}\int_0^{nL}(\eta''(x)^2dx) + \frac{1}{2}\sum_{i=1}^n(k_i\eta(iL)^2)}{\sum_{i=1}^n\int_0^{iL}(\eta'(x)^2dx)}\\ =&\frac{EI\int_0^{nL}(\eta''(x)^2dx) + \sum_{i=1}^n(k_i\eta(iL)^2)}{\sum_{i=1}^n\int_0^{iL}(\eta'(x)^2dx)}\end{aligned} \tag{3.19}$$

where P^{syst3} is the buckling load acting on each floor level, n is the number of evenly distributed bracings, L the distance between the bracings, k_i the stiffness of the i:th bracing, E the modulus of elasticity and I the moment of inertia of the column.

For the particular case of System 3 in which $n = 2$ (number of bracings), the solution of Equation 3.19 would develop in accordance with Equations 3.20-3.23 below.

The assumed shape function:

$$\eta(x) = \Delta_\eta sin(\frac{\pi x}{cL}) \tag{3.20}$$

The number of bracings:

$$n = 2 \tag{3.21}$$

The assumed expression for the variation of the bracing stiffness:

$$k(x) = \frac{3EI}{x^3} \tag{3.22}$$

Inserting Equations 3.20-3.21 into Equation 3.19 gives Equation 3.23 below:

$$\begin{aligned}&\Rightarrow P^{syst3}_{n=2}\\ &= \frac{32L^6EI\pi sin(\pi/c)cos(\pi/c)(2cos(\pi/c)^2-1) + 24cL^4EIsin(\pi/c)^2 + 3cL^4EIsin(2\pi/c)^2}{8cL^7(2-cos(\pi/c)^2-cos(2\pi/c)^2)}\end{aligned} \tag{3.23}$$

Non-linear analysis

In order to also study the strength requirements of the columns (not only viewing them as elastic, as was done in the sections above) and investigate how they are affected by different imperfection shapes and bracings, non-linear incremental analyses are normally required. Paper 2 investigated how slender columns and their bracing requirements were affected by the shape of the imperfections. This was investigated by the use of the commercial finite element program Abaqus. In the paper it was shown that the results are highly sensitive to the input regarding the shape of the imperfections. Most importantly, different shapes should be used for obtaining conservative results for the bracing requirements and for the load-bearing capacity of the column itself. For a conservative estimate of the bracing requirements, the imperfection shape should have its maximum value at the brace points. For a conservative estimate of the strength requirements of the column itself, the maximum value of the imperfection shape should instead be located between the bracings.

Other instabilities

Other important instabilities for columns, not dealt with in the thesis, include twist modes (that can occur, for instance, when a bracing is attached to only one side of the cross-section), local buckling (e.g. shear buckling of webs) and combinations of different buckling types.

3.3 Beams

Generally, lateral torsional buckling of beams is a more complicated topic than column buckling, due to the fact that it involves both bending and torsion (see Figure 3.13), whereas most cases of column buckling involve primarily bending around only one axis [16, 21].

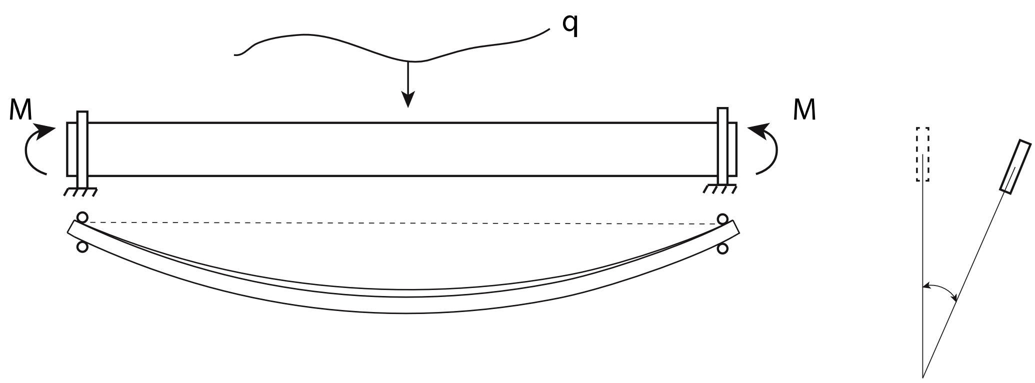

Figure 3.13: *The principle of lateral torsional buckling of a beam.*

The critical moment of a beam can be solved analytically for certain simple cases by means of beam theory equations [4]. For instance, for the beam shown in Figure

3.13, when loaded only by end moments M, (q is assumed to be zero), M_{cr} will, according to Equation 3.24, be:

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_\eta GJ(1 + \frac{EC_w\pi^2}{GJL^2})} \tag{3.24}$$

where EI_η is the bending stiffness (weak axis), GJ the torsional rigidity, EC_w the warping rigidity and L the span.

As can be seen in Equation 3.24, the span L between the restraint points is important in terms of the buckling capacity M_{cr}, this being why bracing along the span is an important method for increasing the buckling capacity. Bracing a beam along its span is similar to reducing the span, i.e. the effective buckling length (the length between successive bracings). In order to illustrate this, Equation 3.24 has been plotted with respect to L in Figure 3.14.

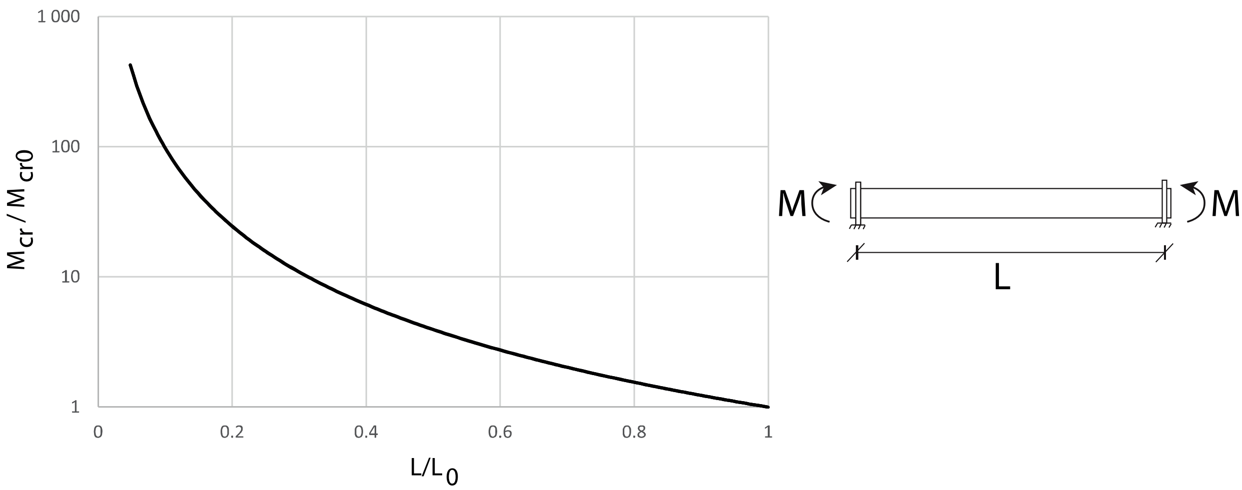

Figure 3.14: *How the critical moment of a simply supported beam loaded with end moments varies with the span L. M_{cr0} is the critical moment when the span is L_0. As can be seen, the critical moment is strongly affected by the span L and thus by the distance between successive bracings.*

The concept of the bracing of beams and the aim of bracing them are basically the same as for columns and will thus not be discussed to such extent here. Winters rigid link method [19] can also be used for beams, the compression flange of the beam being considered as an equivalent column. Further information regarding the bracing of beams can be found in Paper 1.

Paper 1 presents various basic principles regarding the requirements that apply to the bracing of beams. Both numerical and laboratory experiments were carried out. It is stated in the article that it could not be verified that an increased bracing stiffness leads to lower bracing forces. However, with use of a different plot design than the one used in connection with the paper in question, this could easily have been shown. For that reason, a numerical analysis using Abaqus was carried out, enabling a better plot to be presented here; see Figure 3.15.

As shown in Figure 3.15, the bracing forces in bracings of slender beams for a given load, generally decrease with increasing bracing stiffness. For stocky beams, however, or for beams with a loading substantially lower than the critical load, the bracing force increases with an increase in bracing stiffness, up to a certain limit, above which it converges to a particular value related to the magnitude of the initial imperfection. The value of the bracing force is also dependent upon the magnitude of the initial imperfection, in the same manner as was mentioned for columns. The larger the initial imperfection is, the larger the bracing force will be. The initial imperfection used in the study presented in Figure 3.15 was $L/1000$, its maximum value being located close to the bracing point.

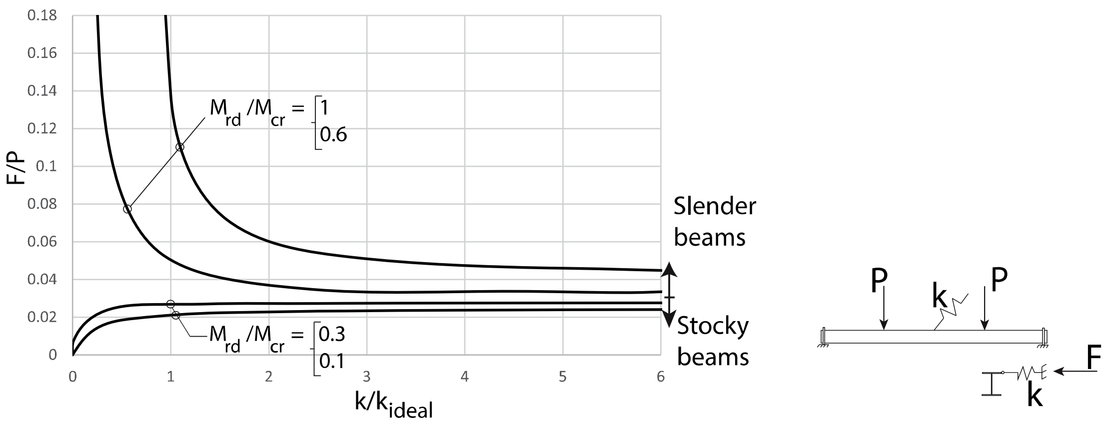

Figure 3.15: *Bracing forces of a simply supported beam braced at mid-span.* M_{rd} *is the moment capacity of the beam, whereas* M_{cr} *is the elastic buckling moment.* P *is the applied load,* k *the current bracing stiffness, and* k_{ideal} *the ideal stiffness.*

In Paper 3, certain specific concerns regarding the properties of the bracing system were also investigated. It was shown in Paper 3 that slip in bracings that can occur is of considerable importance in regard to the expected buckling capacity of the braced beam (and trusses). Further information regarding slip is to be found in Section 3.1.

4 Summary of appended papers

Paper 1

Slender timber beams subjected to gravity loads may buckle in the out-of-plane direction. Normally, the same bracing system that is used to prevent lateral movements of the beams, caused by external transversal loading such as wind, also serves to increase the buckling resistance of the beams. For the idealized case of a perfectly straight beam with bracing, no force develops in the braces since the lateral displacement remains zero. However, since real beams are not perfectly straight, bracing forces develop during loading. This paper describes experimental and analytical studies performed on discretely braced and slender glue-laminated-beams subjected to gravity loads. In particular, the effects of relevant parameters such as i) bracing stiffness, ii) bracing position and iii) shape and magnitude of initial imperfections are reviewed. The results indicate i) that an increase in bracing stiffness leads to an increase in bracing forces (for stocky beams), ii) that a beam should be braced on the compression side in order to maximize the effects of the bracing, and iii) that an increase in the magnitude of the initial imperfections increases the need for bracing.

Paper 2

Finite-element-based programs can be used to design both columns and their bracing systems. As is well known, however, the output obtained is highly dependent upon the input. In the present study, the effects of imperfections on the predicted strength and stiffness requirements of the steel columns involved and of their bracing systems are investigated. Two different systems are analyzed: 1) a braced non-sway column and 2) a braced sway column. It was found that a poor choice of the shape of imperfections can lead to unrealistic results in terms of both the buckling load on the columns and the predicted reactions of the bracings, and that superimposing different imperfection shapes can contribute to obtaining realistic and trustworthy results. It was also shown that the shapes both of the initial imperfections that lead to the lowest buckling loads and of those that result in the strongest forces being directed at the bracings are generally not the same.

Paper 3

This study investigates the consequences of potential slip in the bracing systems of timber members of two different kinds. The first case is that of a simply supported timber beam braced at one point at mid-span. The second case is that of a truss braced at multiple points along its top-chord. A potential source of slip in the bracings can be that of there being oversized holes in the connections. It was shown in the study that slip in the bracing system can reduce the load-bearing capacity of the braced members both through the stresses being greater the elastic limit being reduced. The study also shows that members less stiff in terms of movements in out-of-plane directions (in the direction of the bracings) are less sensitive to the occurence of slip in their bracing systems.

5 Conclusions

The conclusions listed here are taken both from the appended papers as well as from the thesis itself.

- Most of the structural failures that occurred in the past were related to major errors made in the design. The same type of errors can be seen in buildings built 40-60 years ago that then failed in the 1970ties, just as in buildings built after 1980 that failed more recently. An example of an error of this sort is a lack of bracing of the compressed side of beams and columns. It is important to emphasize the significance of learning from the past in order to avoid the repeating of errors. This can for example be achieved through the education of engineers and through use of databases in which information regarding collapses is open to everyone.

- Numerical analyses and laboratory tests on slender glue-laminated timber beams have showed how effective bracing is in reducing lateral displacements in loading (Paper 1). It was found that an increase in bracing stiffness leads to an increase in the bracing force in the case of loads much less than the critical load. This could be due to a greater degree of stiffness attracting greater force in that region of loading. It was also shown that an increase in the magnitude of the initial imperfections results in an increase in the need of bracing.

- It was shown in Paper 2 that a poor choice of an imperfection shape can lead to unrealistic results when nonlinear FE analyses of slender columns are carried out. Examples of imperfection shapes that lead to unrealistic elastic buckling modes were demonstrated. These conclusions probably also hold for other structural elements.

- A method in which imperfections are assessed by superimposing a number of different buckling mode shapes was shown to be rather successful in terms of generating realistic results when non-linear analyses are carried out (Paper 2). The specific approach proposed here defines one particular mode shape as being a major imperfection and a number of other modes as being minor disturbances occurring on top of that one. With this approach, the appearance of an imperfect shape is clearly defined by the major shape, making the approach easy for engineers to employ. The minor shapes do not affect the appearance very much, yet they are highly important in enabling the element in question to buckle in the correct manner, given bracing properties that are present, this serving to ensure that the behavior found is realistic. Eurocode 3, e.g., states that the worst combinations of imperfections should be used in the design. The results obtained in Paper 2 provide guidance concerning how that can be realized in practice.

- In Paper 2 it was shown that an imperfection shape that generates a large initial displacement between the bracings led to higher bending moments in the column at loading than an imperfection shape did for which the maximum initial displacement was at the bracing points. This means that an imperfection shape having its maximum amplitude between the bracings is needed when a conservative estimate of the column strength is aimed at. At the same time, an imperfection shape generating a large initial displacement at the bracing points was found to be associated with there being a higher level of forces in the bracings at loading. Such an imperfection shape should thus be used in order to obtain a conservative estimate of the required bracing strength and stiffness. Accordingly, different imperfection shapes should be used for a conservative design of the bracing system and of the columns, respectively.
- In Paper 3 it was shown that slip in bracing systems has the potential of reducing the expected load-bearing capacity of braced structural members. For a truss braced at several points along its top-chord, the expected buckling capacity was found to be reduced significantly by a slip in the bracing system occurring. Such a slip also led to there being higher stresses in the members due to greater bending around the minor axis. Interestingly enough, it was shown that larger initial imperfections made the members less sensitive to slip. Large imperfections are normally not tolerated, yet for other reasons, such as those of aesthetics and functionality. The same conclusions probably hold as well for other structural elements, such as columns.

6 Further research

Further research which is needed includes other types of buckling than those investigated in connection with the thesis (such as torsional buckling and local buckling), as well as other types of structures (such as arches and frames) and questions of how different elements in a structure interact in connection with buckling (such as in the case of global buckling).

Three important areas of activity that call for further research are the following:

1. In view of the fact in looking at structural failures that occurred in the past, it clearly appears that the most common causes of failure are those related to ignorance and carelessness rather than to lack of research, one can note that one such area of activity is that aimed at determining, in a scientific manner, how different engineers deal with stability-related problems. This can involve determining different engineers' level of theoretical knowledge, and how they use different design tools. Such activity can lead to proposals concerning both improvements in the education of engineers and possible post graduate courses for engineers who design slender structures in their profession. Such activity can also be aimed at assessing whether ”third-party” reviews of construction drawings or of other design-related documentation would be an effective method of discovering design errors and thus preventing future potential building collapses. Today there is no requirement in Sweden for the external review of design documents of buildings (not the case for bridges though). A method of investigating both the state of knowledge of engineers generally and the potential benefit of third party review could be to let a number of engineers review blueprints from a number of different projects, some of these blue-prints being of buildings that had collapsed and others of buildings that had been more adequately designed.

2. In view of its having been shown in the present investigations that the buckling capacity of a slender member is highly affected by both the stiffness of bracing systems an the imperfection shapes these possess, another area of research activity needed would be that of measuring both the stiffness of different common details in existing buildings and the imperfections these possessed, as well as conducting laboratory tests on certain common structural elements. For example, as shown in Paper 3, slip in bracing systems has the potential of reducing the expected buckling capacity of the braced elements involved. Thus, measurements that can either confirm or contradict such slip in real structures

would be valuable as future design input. The results of this activity could be used to refine the rules contained in the building codes.

3. Still another activity that can be seen as important is that of developing a method that can be used to measure or evaluate the stiffness of the bracing system both in already existing buildings and buildings that are being constructed. A trend on the building market would appear to be to build with use of more slender structures, perhaps partly due to hard competition between the different entrepreneurs. At the same time the more slender a structure is, the more prone it is likely be to become instable. It is thus important to develop a method of the type just referred to so as to be able to build slender structures and at the same time ensure that they possess sufficient stability. A methodology that describes such a process could, for example, be implemented in the building code.

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